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# Using river locks to teach hydrodynamic concepts

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## Abstract

In this work, the use of a river lock as a non-formal setting for teaching hydrodynamical concepts is proposed. In particular, we describe the operation of a river lock situated at the Sobradinho dam, on the São Francisco River (Brazil). A model to represent and to analyse the dynamics of river lock operation is presented and we derive the dynamical equations for the rising of the water column as an example to understand the Euler equation. Furthermore, with this activity, we enable the integration of content initially introduced in the classroom with practical applications, thereby allowing the association of physical themes to content relevant in disciplines such as history and geography. In addition, experiences of this kind enable teachers to talk about the environmental and social impacts caused by the construction of a dam and, consequently, a crossover of concepts has been made possible, leading to more meaningful learning for the students.

(Some figures may appear in colour only in the online journal)

## 1. Introduction

According to the National Curricular Parameters (*Parâmetros Curriculares Nacionais*) [1] of Brazil, a student's formation must have, as its main objective, the gaining and comprehension of basic concepts, as well as the scientific preparation and capacity to use different technologies related to different actuation areas. At the secondary level, a most general formation is proposed, which is the opposite to that of a specific one. The development of the capacity to research, analyse, and select information is also proposed. Furthermore, education must aim to develop the capacity to learn, create, and formulate, instead of being a memorization exercise.

In this way, one cannot restrict education to student institutions—it can happen in many other places [2, 3]. Moreover, it is known that a student's knowledge is obtained not only

from experiences occurring inside a classroom, but also from their experiences in everyday life and didactic activities, which can be proposed, for example, by a teacher. Thus, scientific dissemination centres, electronic media, and science museums are important tools in the learning process [4]. According to Langhi *et al* [5], learning may occur in several environments and these can be classified as formal, informal, and non-formal settings. Furthermore, we can cite, as a learning activity, activities known as scientific popularization.

Langhi *et al* [5] define formal education as that occurring in a school environment and other teaching establishments, with its own structure and planning, whose objective is to work, didactically, with systematized knowledge. Non-formal education is that having a collective character and involving educative practicals which do not occur in a scholar environment and have no legal requirements. During these activities, the student experiences the freedom to choose the concepts to be learned. Among the examples offering non-formal settings, one can cite: museums, media, training agencies for specific social groups, and non-conventional teaching institutions, which organize events such as free courses, science fairs, and scientific meetings [5]. It is important to say that, despite the fact that it does not occur in a scholar environment, non-formal education is not free of a certain degree of intentionality and systematization.

In this context, museums and science centres, which can be classified as non-formal education settings [5], can favour conceptual expansion and refinement in an environment which can cause emotions to become coupled with the cognitive process. This has been denoted as an intrinsic motivation for the learning of science [6]. Furthermore, non-formal settings are places providing an appreciation and understanding of sciences through voluntary and individual actions, thereby popularizing scientific and technological knowledge [7]. Nowadays, there is an increasing interest in both the affective and emotional impacts and in the production of meaning and knowledge construction [8].

Given the importance of non-formal settings for a student's learning, there are several works dedicated to researching the influence of museums on science education (see, for example [2, 3, 7, 9, 10]). Nevertheless, the presence of museums and science centres is not a reality in most of the northeast of Brazil. Thus, to visit one of these non-formal spaces can be a tiring, stressful and expensive activity due the distances between the several cities located in this region and the nearest scientific centres. In this case, in order to provide different learning environments for the students, a teacher could propose accessible activities in their local area to arouse the enthusiasm of students for scientific knowledge.

In terms of hydrodynamics teaching, several works have investigated the learning process and new ways to address the concepts related to this issue. For example, one can cite some elaborate experiments designed to teach fluid dynamics [11, 12]. Furthermore, Arellano *et al* have proposed the injection of particles or bubbles in order to simplify the complex task of teaching the hydrodynamics of a swimmer's propulsion to undergraduate students of physical education. With this technique, the students have the opportunity to see how the water is actively moving when the body is propelled through the water [13]. In addition, from the analysis of 15 original simulations, created with GeoGebra software, Romero *et al* [14] devised a questionnaire on the interest of using simulations to teach fluid mechanics to simulation-taught students and compared the answers to those given by students taught without use of simulations. At the examination, the average grades and the percentage of passed students were higher in group 1 than in group 2. However, the author recognises that additional strategies need to be adopted aiming to help students develop the skills required to succeed in a physics course. An apparent paradox in communicating vessels systems is discussed in the work of Miranda [15], in which the author shows that, for a liquid in any connected vessel system, it is not possible to realize Pascal's principle, mass and energy conservation simultaneously.

In addition, there are propositions of hydrostatic teaching from experimental activities using low cost materials, e.g., plastic bottles [16] and water cups [17]. Finally, the transport of water from the roots to the crown of trees is discussed for two conduit architectures in the work of [18] and it is proposed to show this to undergraduate students in order to get an interdisciplinary communication with biology. In this work, the author considers a subject of broad interest because it provides a naturally occurring example of an unusual metastable state of matter.

In this paper, we report the proposition to study the basic concepts of hydrodynamics from a visit to a river lock, which consists of a work of hydraulic engineering that allows boats to ascend or to descend rivers and seas in places where there are gaps (dams or waterfalls). In our case, we have chosen to visit Sobradinho river lock, as it is the nearest structure of this kind to Senhor do Bonfim-BA, Brazil. Furthermore, we show how to construct a model of a river lock in order to study the operation of this system in more detail. We also discuss the opportunity that must be given to the students to discuss and to couple knowledge from other disciplines, such as geography and history, to physics, and to see science as a human activity; thereby gaining an understanding of the social, economic and political impacts arising from the construction of a dam.

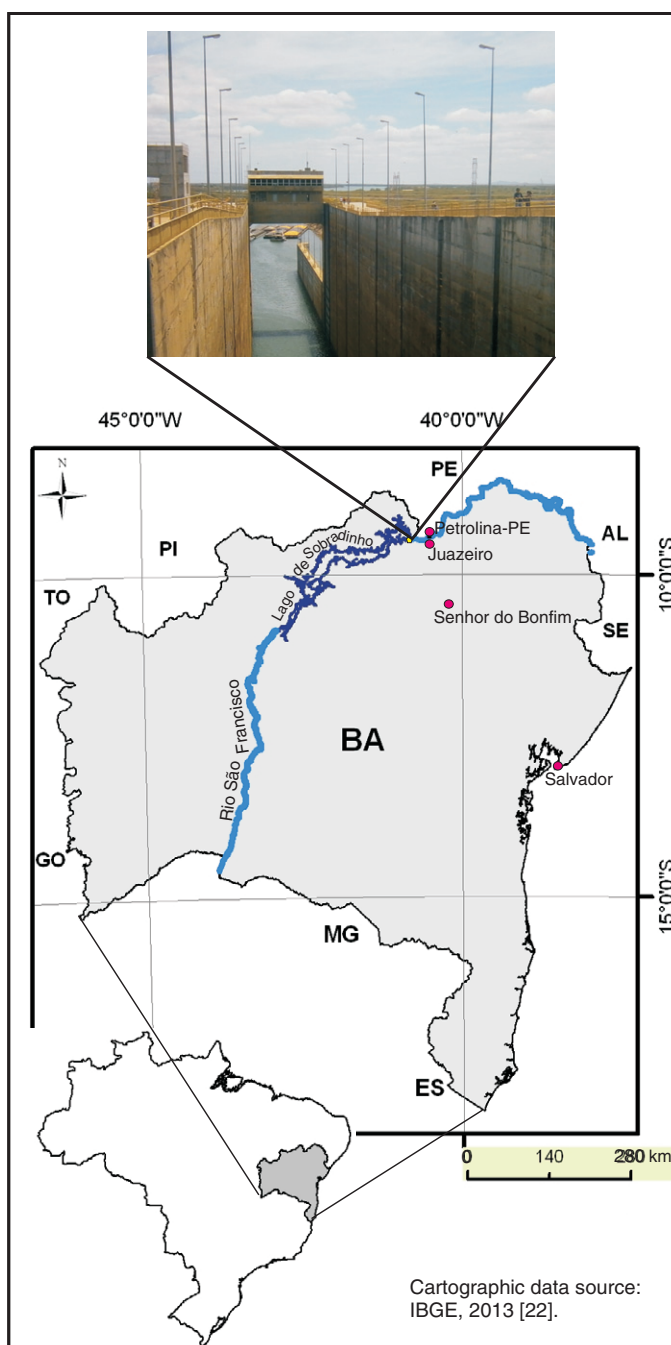
This work is divided as follows: in section 2, we present Sobradinho river lock and talk about its operation and economic importance; section 3 brings a discussion on some hydrodynamic concepts that can be addressed from the operation of a river lock; in section 4, we present the method of constructing a model of a river lock in order to study, in more detail, the river lock's operation; finally, in section 5, the conclusions are presented.

## 2. The Sobradinho dam and the hydrodynamics of river locks

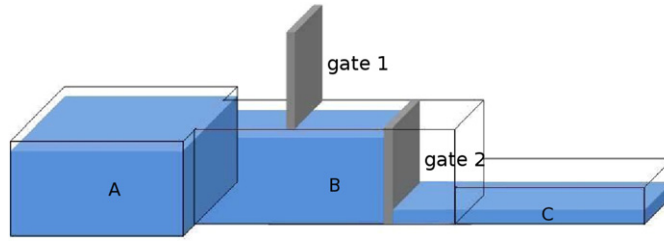
The artificial lake formed from the Sobradinho dam has a length of 320 km [19] (from the municipality of Sobradinho to the municipality of Pilão Arcado) and a water surface area of 4214 km<sup>2</sup>. Its storage capacity is around 34.1 million litres, making it the second largest artificial lake in the world. It ensures, through a depletion of up to 12 m, together with the Três Marias Reservoir, a regulated flow of 2060 m<sup>3</sup> s<sup>-1</sup> during the dry season, allowing operation of all hydroelectric plants at the *Companhia Hidroelétrica do Vale do São Francisco*, situated along the São Francisco River.

The dam incorporates a river lock, owned by *Companhia Docas do Estado da Bahia*, whose camera has 120 m of length and 17 m width, allowing the boats to overcome the gap created by the dam, around 32.5 m, with a maximum filling time of 16 min. This river lock ensures the continuity of traditional navigation between the stretch of the São Francisco River between the cities of Pirapora-MG and Juazeiro-BA/Petrolina-PE (1371 km navigable) enabling waterway transport and, therefore, commercial navigation in the Old Chico (which is how the river is known in the region). In figure 1, one can see a highlighted view of the river lock under discussion. In this same figure, we show a map locating the Sobradinho dam in the state of Bahia-Brazil.

River locks function as stairs or elevators for ships or boats, in which there are two gates separating the two river (or sea) levels. In figure 2, we show a schematic view representing, in a very simplified form, the operation of a river lock. When the boat travels up the river, it enters the lock at the downstream side (marked in the figure as C) and remains in the chamber (region B). The downstream is then closed and the chamber filled with water, causing the boat to rise until it reaches the level of the upper reservoir. Thereafter, gate 1 can be opened and the boat leaves the lock, going into the dam, marked in the figure as region A. When the boat travels down the river, it enters the chamber at the upstream side of the lock and the gate is closed, emptying the chamber gradually until it reaches the level of the lower reservoir.



**Figure 1.** Location of the Sobradinho dam. Featured, a view of the river lock in the dam. The vessel is raised to the highest level by opening a gate, which releases the entry of water coming from the dam in the region of the lock.



**Figure 2.** Schematic view of the operation of a river lock. In static conditions, the pressure must be the same for points situated at the same height; if we open the link between A and B, there will be a water flow from region A to region B until the water level is equal in the two sides. The same argument must be used when we open the link between B and C. In this case, when the board comes from C, going to A, the link between B and C must be opened in order to decrease the water level in B. The gate is opened and the boat can go to B. Now, the link between A and B is opened and the water level in B rises. Gate 1 is opened and the boat can pass to A.

Finally, gate 2 is opened and the boat leaves the river lock. The operations of filling and emptying the chamber are usually made by gravity with the help of small gates and valves. An animation showing the operation of the Sobradinho river lock can be found on the website cited in [19].

### 2.1. Static

In order to study the hydrodynamical principle of the operation of a river lock, we will simplify it to a communicating vessels system, whose dynamical properties can be well defined from the Euler equation [20, 21]:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla p}{\rho} + g \hat{\mathbf{z}}, \quad (1)$$

where  $\mathbf{v}$  is the fluid velocity,  $\rho$  is its density,  $p$  the pressure of the fluid, and  $g$  is the gravity acceleration. For a fluid at rest, we have:

$$\nabla p = \rho g \hat{\mathbf{z}}. \quad (2)$$

If the fluid density is considered constant along its volume and the  $z$  axis is taken as vertical, the equation (2) can be integrated to give:

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0, \quad \frac{\partial p}{\partial z} = -\rho g. \quad (3)$$

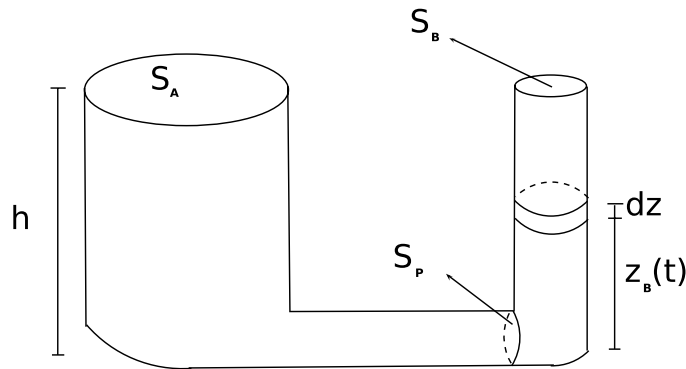
Thereby,

$$p = -\rho g z + \text{constant}. \quad (4)$$

If the fluid has a free surface, in the height  $h$ , for which an external pressure  $p_0$ , at all points, is applied, the surface must be a horizontal plane  $z = h$ . From the condition  $p = p_0$  when  $z = h$ , we have that the constant, in the equation (4), is given by  $p_0 + \rho g h$ , such that:

$$p = p_0 + \rho g (h - z). \quad (5)$$

In this way, given a point in a fluid, the pressure on this point will depend only on the height of the liquid. In this case, from observing figure 2, we can conclude that, from the opening of gate 2, a point situated in region C will be subject, initially, to a pressure lower than the pressure on a point, at the same height, situated in region B. From equation (5), if the regions are linked, the pressure must be equal in the two points. Thus, there will be a



**Figure 3.** Simple model to analyse the rising velocity of the liquid column in a river lock. Here, we have considered that the water flow is a function of the height  $z$  inside region B represented in the figure 2. Once the pressure in the opening linking regions A and B increases with  $z$ , the velocity of the exit of water in this opening diminishes, thus, the liquid column rises with a decreasing velocity.

transference of fluid from region B to region C when the connecting valve between these two regions is opened. The water level in region C does not rise because the water goes into the river, and, in the case of the Sobradinho dam, it goes to the cities of Petrolina/Juazeiro. Then, there is a reduction in the water height in region B until it reaches the level of the river (region C).

From the same principle, when region B is at the same level as the river (C), there will be a flux of water from region A to region B, filling this region and increasing the water height.

## 2.2. Dynamics

Now, we will analyse the rising velocity of the liquid column inside the river lock in the function of the water flow released by the connection valve between regions A and B. Obviously, if we maintain a constant flow  $Q$ , given by  $Q = \frac{dV}{dt}$ , where  $V$  is the volume of water that passes from region A to region B, a boat will rise until reaching the top of the river lock with a constant velocity, given by:

$$v(t) = \frac{Q}{S_B}, \quad (6)$$

where  $S_B$  is the surface area of region B. In this case, the height will be a linear function of the time, given by:

$$z(t) = \frac{Q}{S_B}t, \quad (7)$$

where we have done  $z(0) = 0$ .

Now, we will study what occurs when the water flow is variable with time, which can be obtained, for example, by maintaining the opening of the valve linking A and B with a constant area  $S_P$ , as shown in figure 3. In this case, when the valve is opened, the water level in region B rises, increasing the pressure of the water on the exit  $S_P$ . Thereby, if we maintain the valve opening with a constant area, the water flow will not be constant once the increasing of pressure in region B diminishes the water flow through the opening that separates the two regions.

As has been said, when the link between A and B is opened, there will be a water flow from region A, with area  $S_A$ , to region B, with area  $S_B$ , passing through the opening with area  $S_P$ . In order to determine the velocity of the water when it passes through the opening, we will use Bernoulli's equation [20]:

$$\frac{1}{2}\rho v^2 + \rho gz + p = \kappa, \quad (8)$$

where  $\kappa$  is a constant. In this case, the fluid's velocity through the opening with area  $S_P$  is given by:

$$\frac{1}{2}\rho v_P^2 + \rho gz_P + p_P = \frac{1}{2}\rho v_S^2 + \rho gz_A + p_A, \quad (9)$$

in which the subscript indices  $P$  and  $A$  represent the surfaces with area  $S_P$  and  $S_A$ , respectively.  $z_P \equiv z(t)$  is the height of the water column when it passes to the region with area  $S_B$ . From the figure 3, we have that  $z_A = h$ ,  $z_P = 0$ ,  $p_A = p_0$ ,  $p_P = p_0 + \rho gz_B(t)$ . Furthermore, we will consider  $S_A \gg S_P$ , then the decreasing velocity of the water in the region A is  $v_A = 0$ . Thus, from equation (9), the velocity of the water through the valve linking regions A and B will be given by:

$$v_P(t) = \sqrt{2g[h - z_B(t)]}. \quad (10)$$

In this way, defining  $z_B(t) \equiv z(t)$ , we have that the water flow  $Q(t)$  through the region with area  $S_P$  is:

$$Q(t) = S_P \sqrt{2g[h - z(t)]}. \quad (11)$$

And, in this case, the rising velocity of the water at the river lock (region B) will be:

$$v_B(t) = \frac{Q(t)}{S_B} = \frac{S_P}{S_B} \sqrt{2g[h - z(t)]}. \quad (12)$$

Once  $z(0) = 0$ , we have that the velocities of the water through the valve,  $v_P(t)$  and in the river lock region,  $v_B(t)$ , in the instant  $t = 0$  are given by  $v_P(0) = \sqrt{2gh}$  and  $v_B(0) = \frac{S_P}{S_B} \sqrt{2gh}$ . Furthermore, they decrease in value when the height of the water column rises, in such way that these velocities will vanish when  $z(t) = h$ .

Finally, with the aim of determining the function,  $z(t)$ , for which the height of the water rises in region B, we start from the velocity definition  $v \equiv \frac{dz}{dt}$ . One can note that:

$$\int \frac{dz}{\sqrt{2g[h - z(t)]}} = \frac{S_P}{S_B} t. \quad (13)$$

This integral is evaluated to give:

$$z(t) = h + \left(\kappa \frac{S_P}{S_B}\right) t - \frac{1}{2}g \left(\frac{S_P}{S_B}\right)^2 t^2 - \frac{\kappa^2}{2g}, \quad (14)$$

where  $\kappa$  is a constant of integration. Taking the initial boundary condition  $z(0) = 0$ , we obtain  $\kappa = \sqrt{2gh}$ , and so:

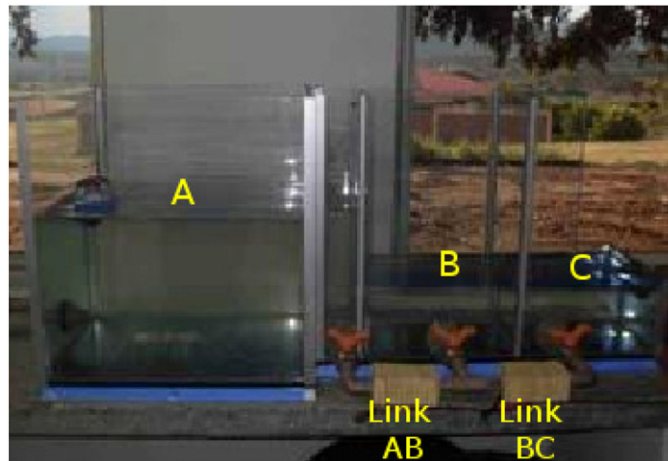
$$z(t) = \sqrt{2gh} \left(\frac{S_P}{S_B}\right) t - \frac{1}{2}g \left(\frac{S_P}{S_B}\right)^2 t^2. \quad (15)$$

One can note that the rising of the water level in the river lock is well represented by a uniformly variable rectilinear motion function, in which the water, and consequently the boat, begins its upward movement with initial velocity  $v_B(0) = \frac{S_P}{S_B} \sqrt{2gh}$ , decreasing its value with a constant acceleration given by  $a = \left(\frac{S_P}{S_B}\right)^2 g$ . In this way, the time needed for the boat to reach the highest point of the river lock,  $z(t) = h$ , and continue its flux, is:

$$t_{\text{up}} = \frac{S_B}{S_P} \sqrt{\frac{2h}{g}}. \quad (16)$$

As expected, the rising time of the boat is directly proportional to the river lock surface area,  $S_B$ , and decreases with the increasing of the height of the river lock.





**Figure 4.** Photograph of the model constructed in order to observe, inside the classroom, the operation of a river lock. Here, we highlight regions A, B and C, represented in the schema of figure 3.

### 3. Constructing a river lock model

The visit to the river lock has been important because, in this environment, the students could have contact with several devices that could not be viewed in a formal space (classroom). Furthermore, they have an opportunity to hear and discuss the theoretical and practical aspects of the operation of a river lock with a professional having experience with the possible problems arising from the operation of these structures. Another advantage of this visit comes from the opportunity to observe the probable social and environmental impacts caused by the construction of a dam.

After the visit to the Sobradinho dam, as a part of a multidisciplinary activity involving several areas of knowledge, as such as physics, history, geography and Portuguese, we have constructed a model of a river lock (see figure 4), in order to evaluate the learning process and systematize the acquired knowledge. The model was constructed according the schema of figure 2. In order to avoid water waste, when it passes from region B to C, we have placed a vessel to receive the water exiting from the model. A water pump returns the water from the vessel to region A.

In figure 4, we present a photograph of the constructed model. In that, one can note the highlighted regions A, B and C. The valves linking the regions have been represented with pipes and taps (link AB and link BC), which release water when opened. When link AB is opened, we release water flow from region A to region B. In this case, the liquid column height in B rises. In this model, in order to ensure the constant height of region A, we have filled the vessel while the water has being flowing to region B. This procedure will ensure that the rising of the water in region B obeys equation (15), that is, the liquid column is rising in an uniformly variable rectilinear motion. Obviously, by closing link AB and opening link BC, the water flows from B to C, however, once the height at region C stops rising, the passage of the water from B to C must obey equation (7), once the flow must be considered constant.

With the aim of testing the validity of equation (15), we performed some measurements with the constructed model. The model's parameters are  $h = 34.5$  cm,  $S_p = \pi$  cm<sup>2</sup>, and  $S_B = 847.9$  cm<sup>2</sup>. To predict the time required to fill B, we used  $g = 9.81$  m s<sup>-2</sup>, obtaining

$t_{\text{up}} \approx 71.58$  s. Besides measuring the filling time, we determined the height of the water column for  $t = 30$  s and  $t = 60$  s. The predicted values are  $z(30\text{ s}) \approx 22.86$  cm and  $z(60\text{ s}) \approx 33.60$  cm. The obtained experimental values were obtained after ten measurements and they were  $z(30\text{ s}) = 21.5 \pm 1.4$  cm,  $z(60\text{ s}) = 31.8 \pm 1.3$  cm and the filling time was  $t = 69.72 \pm 0.7$  s. The disagreement among the theoretical and experimental values is associated with errors due to sealing problems in the gates, which made it possible that there was a slightly higher water flow than expected from region A to B. In addition, there may be variations due to the actual value of the local gravity acceleration.

#### 4. Conclusions

We have proposed a visit to a river lock as a non-formal setting for teaching hydrodynamics. In particular, we have visited the Sobradinho dam, situated on the São Francisco river, 150 km away from Senhor do Bonfim. The main objective of this excursion was to show the operation of a hydroelectric plant and, in particular, a river lock.

We believe that this activity can enable the integration of propaedeutic content, learned in the classroom, and its practical application. We discussed the physical principles based on the operation of a river lock, as well as its economic importance, in such way that students might realize that physics is not a subject disconnected from reality and instead has important links with other areas of knowledge. Thus, students realized that economic development is linked to the scientific development of a country. When we returned to a formal space (classroom), we constructed a model to show, in more detail, the operation of a river lock for the students.

The visit to the river lock, with the participation of students and teachers, allowed a greater integration among stakeholders and the contextualization of contents relevant to each discipline. As a consequence, it was accomplished with an interdisciplinarity among physics, history and geography, where it has been possible to discuss problems such as siltation, transportation, economy, water use for irrigation and electricity generation, as well as the social, economical and environmental impacts of the construction of a dam. Furthermore, the history teacher could explain the evolution of the economy and culture of the communities located in the places affected by the Sobradinho dam.

Finally, besides the river lock, the presence of a hydroelectric plant at Sobradinho gave us the opportunity to discuss other physical concepts with the students, e.g., the processes of electric power generation, hydropower, alternative forms for energy generation, and electromagnetic induction. We could review the energy conservation principle.

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